Math 105 – Thonney Name: ___________________________________

Activity F3

Practice: Annuities

1. Amina recently set up a retirement account to save for her retirement. She arranged to have \$50 taken out of each of her biweekly (every two weeks) checks; her account will earn 7% interest. She just had her 30th birthday, and she plans to retire at age 65. Find the following:

   a) The balance in her account at the end of the 35 years.

   d=50, r= 0.07, k = # compounding periods per year = 26, N = # years = 35

   $$P_N = \frac{d\left(\left(1+\frac{r}{k}\right)^{(N\cdot k)}-1\right)}{\left(\frac{r}{k}\right)} = \frac{50\left(\left(1+\frac{0.07}{26}\right)^{(35\cdot 26)}-1\right)}{\left(\frac{0.07}{26}\right)} = \$195,933.38$$ This is the amount in her account at the end of 35 years.

   (note: $N\cdot k = 35\cdot 26$ is in the exponent. It's not just multiplied.)

   b) Amina's total contribution to the account.

   Her contribution is \$50 $\cdot$ 26 contributions per year $\cdot$ 35 years= \$45,500 total contributions.

   c) The total interest Amina will earn.

   The total interest she earns is the difference between the two answers above.
   \$195,933.38 - \$45,500 = \$150,433.38

2. Mr. and Mrs. Abassi set up an IRA to save for their retirement. They will deposit \$250 from each of Mrs. Abassi's semi-monthly (2 times per month) paychecks, and their account will earn 5% interest.

   a) How much will they have in their account after 30 years?

   d=\$250, r = 0.05, k = 24 (2 times per month = 24 times per year), N = 30 years

   $$\frac{250\left(\left(1+\frac{0.05}{24}\right)^{(30\cdot 24)}-1\right)}{\left(\frac{0.05}{24}\right)} = \$416,964.19$$ This is the total amount they will have after 30 years.

b) After 30 years, the Abassis retire. They convert their annuity to a savings account, which earns 4% interest compounded monthly. At the end of each month, they withdraw $1800 for living expenses. Complete the chart below for their post-retirement account.

Month Number	Account Balance at Beginning of the Month	Interest for the Month	Withdrawal	Account Balance at the End of the Month
1	$416,964.19	=416,964.19*(0.04/12) $1389.88	1800	=416,964.19 +1389.88-1800 =$416,554.07
2	$416,554.07	$1388.51	1800	$416,142.58
3	$416,142.58	$1387.14	1800	$415,729.72
4	$415,729.72	$1385.77	1800	$415,315.49
5	$415,315.49	$1384.38	1800	$414,899.87

c) What will happen to their account balance if they continue this process?

They will slowly use up their savings.

3. Jeanne and Harold Kimura want to save $750,000 for retirement. Thanks to their employers, they have access to a tax-deferred retirement account that earns 8% interest, compounded monthly.

a) If they have 40 years to save, how much do they need to deposit each month?

$P_N = 750,000$ d = ? r =0.08 k = 12 N = 40

$$750,000 = \frac{d\left(\left(1+\frac{0.08}{12}\right)^{(40\cdot 12)} - 1\right)}{\left(\frac{0.08}{12}\right)}$$

750,000 = d · 349.0078...

d = 750,000 ÷ 349.0078... = $214.84 This is the required monthly deposit amount in order to meet their goals.

b) How much of the $750,000 they will have will be from contributions? From interest?

Total contributions = $214.84 *12 payments a year*40 years = $103,123.20

Interest = $750,000 - $103,123.20 = $646,876.80

c) The Kimuras are interested in retiring early. How much would they have to deposit each month to reach the same goal in just 25 years?

$$750,000 = \frac{d\left(\left(1+\frac{0.08}{12}\right)^{(25\cdot 12)}-1\right)}{\left(\frac{0.08}{12}\right)}$$

750,000 = d $\cdot$ 951.026...

d=750,000 $\div$ 951.026...

d=$788.62 This is the monthly contribution they would need to make in order to retire early.

d) If they retire in 25 years as in part (c), how much of their account balance is from interest?

total contributions = $788.62*12*25 years = $236,586

interest = 750,000 - $236,586 = $513,414

e) The final question they have is whether $750,000 is "enough" for retirement. Identify what pieces of information you would need in order to be able to answer this question for them.

Let's say that they were originally planning to retire at age 65 (that's 40 years after age 25, when potentially they could have started saving). If they decide to retire only 25 years after they start saving, that means they retire at age 50 instead.

Living until age 85 means they both need to support themselves for 35 years after retirement. That $750,000 has to cover more years of expenses. $750,000 ÷ 35 years = $21,429/year, or $1786 per month. This amount is what both of them would be sharing.

It's possible they would have social security, or a part time job, or other sources of income. Do you think this would be enough? It might depend on whether or not they have a mortgage or rent payment.

Practice: Payout Annuities & Loans

Shelby and Natalie Kersting want to buy a house. They are looking at a house that costs $145,000, and they will also need to pay 3% of the house purchase price in closing costs.

1. Find the total amount of money they will borrow if they finance the entire cost plus closing costs:

   They will need $145,000*1.03 = $149,350. (Closing costs are $4350.)

The Kerstings have several options for a mortgage (home loan). They are trying to decide between two options: a 15-year mortgage that has an annual interest rate of 4.5%, or a 30 year mortgage with an annual interest rate of 5.5%. *(Banks tend to have higher interest rates on longer loans due to risk.)*

2. Let's first explore option 1, the 15-year mortgage with an annual interest rate of 4.5%.
   a) How much would their monthly payment (principal and interest) be?

Use the loan formula $P_0 = \frac{d\left(1-\left(1+\frac{r}{k}\right)^{(-N\cdot k)}\right)}{\left(\frac{r}{k}\right)}$. P_0=$149,350 d = ? r = 0.045 k = 12 N = 15

So $149,350 = \frac{d\left(1-\left(1+\frac{0.045}{12}\right)^{(-15\cdot 12)}\right)}{\left(\frac{0.045}{12}\right)}$ which means 149,350 = d*130.72 so d = 149,350 ÷ 130.72
d=$1142.52 This is their monthly payment.

   b) How much would they pay in all, over the course of the loan?

They would pay $1142.52*12 payments per year * 15 years = $205,653.60 total payments

   c) How much of your answer to part (b) is interest?

The loan was for $149,350 but the total they paid was more: $205,653.60. The difference between these two is interest. $205,653.60 - $149,350 = $56,303.60 is the total amount of money spent on interest.

3. Now let's explore option 2, the 30-year mortgage with an annual interest rate of 5.5%.
   a) How much would their monthly payment be?

P_0=$149,350 d = ? r = 0.055 k = 12 N = 30

$$149,350 = \frac{d\left(1-\left(1+\frac{0.055}{12}\right)^{(-30\cdot 12)}\right)}{\left(\frac{0.055}{12}\right)}$$

149,350 = d*176.12…
d=149,350 ÷ 176.12…
d=$847.99 This is a much lower monthly payment compared to the one in question 2a, but they will be paying for twice as long.

   b) How much would they pay in all, over the course of the loan?

$847.99 * 12 payments per year * 30 years = $305,276.40 total.

   c) How much of your answer to part (b) is interest?

The interest is the difference between the amount paid and the amount borrowed.
$305,276.40 - $149,350 = $155,926.40 is interest.

4. Which option costs less in interest? How much interest does that option save?

The 15 year loan costs \$56,303.60 in interest. The 30 year loan costs \$155,926.40 in interest. Taking the 15 year loan saves \$155,926.40 - \$56,303.60 = \$99,622.80.

5. What are some advantages of taking the option that costs more in interest?

Having a lower monthly payment puts less pressure on you, and you are likely to be able to keep up with the payments. Extra money is also available for home improvements.

6. Suppose the Kerstings could save the extra money they save each month by taking the lower monthly payment.

a) How much would they be saving each month? (What is the difference between the monthly payment options?)

\$1142.52 - 847.99 = \$294.53 is the difference between the monthly payments in the two scenarios.

b) If they can save that money in a money market account earning 7% per year (annually, after taxes), how much would they have at the end of 30 years? *(Hint: you will need a different formula than you used for the loan calculations.)*

Use d = \$294.53, r = 0.07, k = 12, N = 30 years and plug into the Savings Annuity Formula:

7. $$P_N = \frac{d\left(\left(1+\frac{r}{k}\right)^{(N\cdot k)}-1\right)}{\left(\frac{r}{k}\right)} = \frac{294.53\left(\left(1+\frac{0.07}{12}\right)^{(30\cdot 12)}-1\right)}{\left(\frac{0.07}{12}\right)} = \$359{,}318.06$$

c) Does this amount of money make up for the extra interest they had to pay?

Yes!

8. Finally, with everything you know about the Kerstings and their options, which option would you recommend for them?

Paying off a loan as soon as possible always sounds good, but if your loan interest rate is lower than an investment opportunity, then it could make sense to keep the 30 year mortgage and invest the monthly difference instead.

Just make sure that the interest rates you are counting on are fixed, not variable interest rates!

Math 105 Name: ___________________________________

Week 4 Activity F5 Answer Key

Finance Review: Which Equation?

With your partner or small group, decide which type of problem it is, then solve the problem.

1. You have $500,000 saved for retirement. Your account earns 5% interest. How much will you be able to pull out each month, if you want to be able to take withdrawals for 20 years?

   Circle: compound interest savings annuity **payout annuity** loans

   This is a payout annuity because there are regular withdrawals.

   $$P_0 = \frac{d\left(1-(1+\frac{r}{k})^{(-N\cdot k)}\right)}{\left(\frac{r}{k}\right)}$$ P_0=$500,000 r = 0.05, k = 12, N = 20 years

   $$500,000 = \frac{d\left(1-\left(1+\frac{0.05}{12}\right)^{(-20\cdot 12)}\right)}{\left(\frac{0.05}{12}\right)}$$

   $500,000 = d*151.53

   d = 500,000 ÷ 151.53 = $3299.78 This is the monthly withdrawal amount possible.

2. You deposit $2000 into an account earning 4% interest compounded monthly. How much will you have in 25 years?

   Circle: **compound interest** savings annuity payout annuity loans

   This is compound interest is because there is a single deposit, not recurring deposits.

   $$P_N = P_0\left(1+\frac{r}{k}\right)^{(N\cdot k)}$$

   P_0 = 2000, r = 0.04, k = 12, N = 25

   $$P_N = 2000\left(1+\frac{0.04}{12}\right)^{(25\cdot 12)}$$ =$5427.53

3. Jose has determined he needs to have $800,000 for retirement in 30 years. His account earns 7% interest. How much does he need to deposit each month to meet his goal?

Circle: compound interest **savings annuity** payout annuity loans

This is a savings annuity because there are recurring (many) deposits. We are given P_N and need to find d.

$$P_N = \frac{d\left(\left(1+\frac{r}{k}\right)^{(N\cdot k)}-1\right)}{\left(\frac{r}{k}\right)}$$

$$800,000 = \frac{d\left(\left(1+\frac{0.07}{12}\right)^{(30\cdot 12)}-1\right)}{\left(\frac{0.07}{12}\right)}$$

800,000 = d*1219.97

d = $655.75 should be deposited each month in order to meet the goal.

4. You want to buy a $10,000 car. The company is offering a 3% interest rate for 36 months (3 years). What will your monthly payments be?

Circle: compound interest savings annuity payout annuity **loans**

This is a car loan. Use the Payout Annuity/Loans Formula.

$$P_0 = \frac{d\left(1-\left(1+\frac{r}{k}\right)^{(-N\cdot k)}\right)}{\left(\frac{r}{k}\right)}$$ P_0=10,000, r = 0.03, k = 12, N = 3 years

$$10,000 = \frac{d\left(1-\left(1+\frac{0.03}{12}\right)^{(-3\cdot 12)}\right)}{\left(\frac{0.03}{12}\right)}$$

10,000 = d*34.386…
d = 10,000 ÷ 34.386
d = $290.81 is the monthly payment.

5. How much would you need to deposit in an account now to have $6,000 in the account in 8 years? Assume the account earns 6% interest. List any other assumptions you make.

Circle: **compound interest** savings annuity payout annuity loans

Sounds like there's a single deposit, so use the compound interest formula. We don't know how often interest is compounded, so you can assume k = 12 (if you assume something else, you will get a similar but slightly different answer.)

$$P_N = P_0\left(1 + \frac{r}{k}\right)^{(N \cdot k)}$$

P_N = \$6000, P_0 = ?, r = 0.06, k = 12, N = 8

$$6000 = P_0\left(1 + \frac{0.06}{12}\right)^{(8 \cdot 12)}$$

6000 = P_0*1.6141… P_0 = 6000 ÷ 1.6141 = \$3717.14

This is what you would have to deposit now to have \$6000 in 8 years.

6. You want to be able to withdraw $8,000 each quarter for the next 5 years. Your account earns 8% interest. How much do you need in your account at the beginning to make this happen?

Circle: compound interest savings annuity **payout annuity** loans

This is a payout annuity because we are talking about regular withdrawals.

$$P_0 = \frac{d\left(1 - \left(1 + \frac{r}{k}\right)^{(-N \cdot k)}\right)}{\left(\frac{r}{k}\right)}$$

d=8,000, r = 0.08, k = 4, N = 5 years

$$P_0 = \frac{8000\left(1 - \left(1 + \frac{0.08}{4}\right)^{(-5 \cdot 4)}\right)}{\left(\frac{0.08}{4}\right)}$$

P_0 = \$130,811.47 is what you would need to start with for this to work.

7. **Challenge question:** Mike plans to make contributions to his retirement account for 15 years. After the last contribution, he will start withdrawing $10,000 each quarter for 10 years. Assuming Mike's account earns 8% interest compounded quarterly, how large must his quarterly contributions be during the first 15 years, in order to accomplish his goal?

We need to figure out the payout annuity part first. Once we have P_0, we can then solve for what the regular contributions to the savings annuity will need to be.

$$P_0 = \frac{d\left(1-(1+\frac{r}{k})^{(-N\cdot k)}\right)}{(\frac{r}{k})}$$

For both, we have k = 4, r = 0.08. For the payout annuity we have N = 10 years and d = 10,000 is the regular withdrawal amount.

$$P_0 = \frac{10{,}000\left(1-\left(1+\frac{0.08}{4}\right)^{(-10\cdot 4)}\right)}{\left(\frac{0.08}{4}\right)}$$

P_0=$273,554.79 is what is needed at the start of retirement.

This becomes the P_N for the savings annuity (his goal as he saves toward retirement). We still have k =4, r = 0.08, but now N = 15 years and we are solving for d. Use the savings annuity formula:

$$P_N = \frac{d\left((1+\frac{r}{k})^{(N\cdot k)}-1\right)}{(\frac{r}{k})}$$

$$\$273{,}554.79 = \frac{d\left(\left(1+\frac{0.08}{4}\right)^{(15\cdot 4)}-1\right)}{\left(\frac{0.08}{4}\right)}$$

$273,554.79 = d*114.05

d = $2398.52 is the needed quarterly contribution